Waveguiding properties of a line of periodically arranged passive dipole scatterers

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Abstract **Electromagnetic properties of line-periodical arrangements of passive loaded dipole scatterers are studied. An analytical solution for eigenwaves propagating along infinite lines of dipoles is presented. Conditions of existence of guided-wave solutions are established. It is shown that in arrays of capacitively-loaded antennas very rapid phase variations along the line are possible, which can possibly be used to realize wide-band superdirective reflectors.**

I. Introduction

Electromagnetics of periodic structures is a very old and well developed field of research. Various spatially-periodic arrangements are used in many practical devices, such as microwave and optical filters, array antennas, lasers. However, this topic remains of current interest. Very much attention in the literature has been recently payed to electromagnetic properties of periodical structures, especially in view of potential applications in light-wave technology. Nowadays, new applications in microwave filters and other devices are discussed. Linear periodical arrangements of small inhomogeneities (small disk patches or small holes in the ground plane) have been recently studied experimentally, with interesting resonance effects revealed [1], [2]. Waveguide channels in photonic crystals can be realized, for example, by removing one line of inclusions. This can be considered as a periodic perturbation of a regular crystal, and treated with similar techniques as other periodic arrangements.

In the antenna theory, electromagnetics of periodical structures is the key of understanding antenna arrays. Field coupling of antennas in an array is usually considered as a parasitic factor since this can cause scan blindness effect. For an array of small dipole antennas field coupling has been studied for example in [3].

Physically, this effect is connected to so called Wood anomalies of gratings, caused by possible excitation of higher-order Floquet modes or surface modes in the array. This is in fact the same phenomenon which is utilized in guided-mode resonant optical filters [4], [5]. In case of antenna arrays, to achieve a superdirective operation excitation of adjacent antenna elements should be out of phase or similar to that. It has been shown [6], [7] that such excitation is possible in arrays of passive resonant scatterers (conductive cavities with slot openings and resonant grooves have been considered).

Here we consider a conceptually simple periodic electromagnetic system in which all practically interesting effects can be realized. We solve the corresponding problem using analytical means and reveal various modes of operation as dependent on the properties of the array elements and the frequency. In this presentation the emphasis is on the waveguide properties of periodical arrangement of small particles along a straight line.

In this study we assume that every inclusion can be modelled as an electric (or, using duality, magnetic) dipole. That is, the geometrical size of every separate inclusion is small compared to the wavelength. For example, these can be short pieces of conducting wires, small dielectric spheres or other similar objects. Loading short antennas by bulk loads (or using coated spheres or coated wire antennas for example), the polarizability can be changed. However, as the electrical size is assumed to be small, the radiation properties are still that of a short dipole antenna. Here we consider the case when all the dipoles are directed along the axis of the structure. No assumption is made regarding the distance between inclusions and the full-wave interaction between all particles is taken into account.

II. Eigenvalue equation and the interaction field

Geometry of the problem is shown in Figure 1: small dipole particles are periodically arranged along a certain axis in space. Electromagnetic parameters of the surrounding isotropic space are denoted by ϵ_0 and μ_0 although the theory is not restricted to the free-space background. Polarizability of every inclusion we denote by α. All the induced dipole moments are directed along the axis z.

A. Eigenvalue equation

For an arbitrarily chosen dipole on the line array (position $z = 0$)

$$\mathbf{p}(0) = \alpha \mathbf{E}_{\rm loc} \qquad (1)$$

where $\mathbf{E}_{\rm loc}$ is the *local* field created by external sources and all the other particles. Assuming that the external

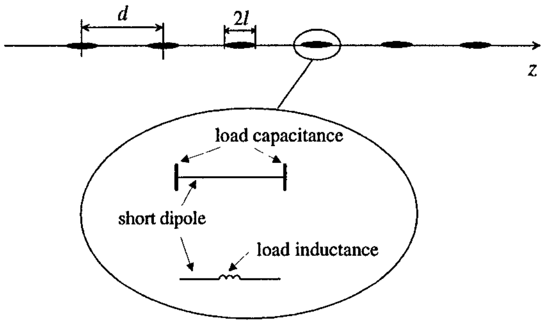

Fig. 1. Periodical arrangement of longitudinally directed dipole particles along a straight line. The system is infinite in the z direction. Each inclusion is an electric dipole (wire antenna) which can be loaded by bulk passive loads. Examples of capacitive and inductive loadings are shown.

field $\mathbf{E}_{\rm ext}$ is a plane wave (or it is absent), we can make use of the Floquet theorem and write

$$\mathbf{p}(nd) = e^{-jqnd}\mathbf{p}(0) \qquad (2)$$

where q is the propagation factor which we will determine in this study. The local field is created by external sources and by the other dipoles in the array:

$$\mathbf{E}_{\rm loc} = \mathbf{E}_{\rm ext} + \sum_{n=-\infty,\ n\neq 0}^{\infty} \frac{1}{2\pi\epsilon_0}\left(\frac{1}{(|n|d)^3} + \frac{jk}{(nd)^2}\right) e^{-jk|n|d} e^{-jqnd}\mathbf{p}(0) \qquad (3)$$

where we have substituted the electric dipole fields (we use the common notation $k = \omega\sqrt{\epsilon_0\mu_0}$).

Let us denote by β the *interaction constant*

$$\beta = \sum_{n=-\infty,\ n\neq 0}^{\infty} \frac{1}{2\pi\epsilon_0}\left(\frac{1}{(|n|d)^3} + \frac{jk}{(nd)^2}\right) e^{-jk|n|d} e^{-jqnd} \qquad (4)$$

With this notation, at the position of the reference dipole $\mathbf{p}(0)$ the local field is

$$\mathbf{E}_{\rm loc} = \mathbf{E}_{\rm ext} + \beta\mathbf{p}(0) \qquad (5)$$

If there is no incident field, then $\mathbf{p}(0) = \alpha\beta\mathbf{p}(0)$. Thus, we have the eigenvalue equation $\alpha\beta = 1$ or

$$\frac{1}{\alpha} = \sum_{n=-\infty,\ n\neq 0}^{\infty} \frac{1}{2\pi\epsilon_0}\left[\frac{1}{(|n|d)^3} + \frac{jk}{(nd)^2}\right] e^{-jk|n|d} e^{-jqnd} \qquad (6)$$

The imaginary part of the interaction constant β can be calculated exactly in closed form, and it can be also found from the energy conservation condition. The real part has to be evaluated numerically, and the corresponding series converges rather quickly.

III. Propagation of waves along lines of dipoles

A. General relations

To study travelling or exponentially decaying waves along the z axis we shall study the interaction constant (4) and the eigenvalue equation (6) in more detail, for the case when $q \neq 0$. If there is some radiation loss of energy and there is energy dissipation in the inclusions, the eigenvalue equation reads (assuming that ${\rm Im}\{1/\alpha\} = {\rm Im}\{1/\alpha_{\rm rad}\} + {\rm Im}\{1/\alpha_{\rm loss}\}$)

$${\rm Re}\left\{\frac{1}{\alpha}\right\} + j\frac{k^3}{6\pi\epsilon_0} + j{\rm Im}\left\{\frac{1}{\alpha_{\rm loss}}\right\} = \beta \qquad (7)$$

This is a complex equation in which the real and imaginary parts should vanish. If the particles have no dissipation, the propagation factor q can be real. For real propagation factors, the last equation takes the form

$${\rm Re}\left\{\frac{1}{\alpha}\right\} + j\frac{k^3}{6\pi\epsilon_0} + j{\rm Im}\left\{\frac{1}{\alpha_{\rm loss}}\right\} = j{\rm Im}\{\beta\} + \frac{1}{\pi\epsilon_0}\sum_{n=1}^{\infty}\left[\frac{\cos knd}{(nd)^3} + \frac{k\sin knd}{(nd)^2}\right]\cos qnd \qquad (8)$$

Because in the region[1] $kd < qd < 2\pi - kd$ the dipole radiation term $k^3/(6\pi\epsilon_0)$ in equation (8) cancels out, propagating guided waves can exist under this condition. Physically, this cancellation comes about because of the energy conservation requirement: the power radiated by each particle equals the power received by the same particle, so that the line does not radiate any power in the far zone.

If the line is lossy, the propagation factor obviously must be a complex number. Guided waves (with some loss of energy) are possible as in other slow-wave structures. The field is confined to the line if k_ρ has nonzero imaginary part.

If q is real but smaller than k, no guided waves can exist in lines of passive scatterers. In this case the imaginary part of the right-hand side of (8) is smaller that of the left-hand side. The power received by every particle (right-hand side) is smaller than that radiated by the same particle (left-hand side) because some part of the power is radiated into a cylindrical wave.

B. Guided waves in lossless structures

For the guided-wave solutions with real propagation factors we have $q > k$ and imaginary transverse wave number k_ρ. In this situation electromagnetic fields exponentially decay in the transverse direction. Existence of guided-wave solutions and the dispersion relation of these waves can be determined from the

[1] Note here that the imaginary part of the interaction constant is 2π-periodic with respect to qd.

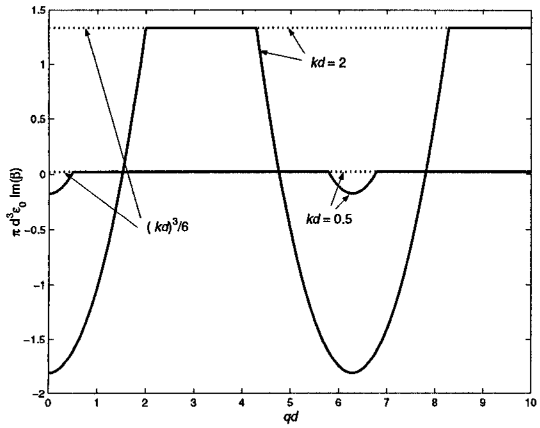

Fig. 2. Normalized imaginary part of the interaction constant for the guided-mode regime.

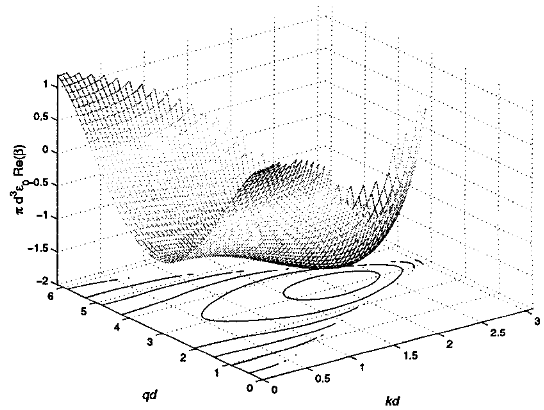

Fig. 3. Normalized real part of the interaction constant for the guided-mode solutions.

corresponding complex eigenvalue equation discussed above.

For lossless particles, the scattering loss of every inclusion should be balanced by the interaction field created by other inclusions. To determine if this is possible, we consider the imaginary part of the eigenvalue equation (8):

$$\frac{(kd)^3}{6} = \sum_{n=1}^{\infty} \left[\frac{kd \cos nkd}{n^2} - \frac{\sin nkd}{n^3} \right] \cos nqd \qquad (9)$$

This series can be expressed in closed form. The result shows that guided-mode solutions are permitted by the energy conservation in the region $kd < qd < 2\pi - kd$ (this is also true in the regions $kd + 2\pi m < qd < 2\pi(m+1)$, $m = 1, 2, \ldots$). This is illustrated by Figure 2, where the regions of the validity of the energy conservation are clearly seen.

Next, let us consider the real part of the eigenvalue equation. The real part of the interaction constant has to be evaluated numerically. The result is shown in Figure 3. Calculations are made only for the region $kd < qd < 2\pi - kd$ where the guided solutions are possible (the imaginary part of the dispersion equation is satisfied). On the floor of the graph, curves of constant levels of the function are shown. These curves show dispersion curves for the guided modes in case if the left-hand side of the real part of the dispersion equation (that is, $\mathrm{Re}\{1/\alpha\}$) is frequency-independent. The family of dispersion curves for this case is also shown in Figure 4. Here, both guided-wave and leaky wave solutions are shown. The two regimes are separated by the line $q = k$. For $q < k$ the solutions to the dispersion equation are leaky modes, because $k_\rho = \sqrt{k^2 - q^2}$ is real and the line of dipoles radiates power. For larger normalized frequencies $kd > \pi$ there are no guided-wave solutions, since the energy conservation requirement cannot be satisfied. This is so because for larger kd there exist higher-order radiating Floquet modes which take power away from the guide.

Thus, depending on the value of the real part of the inverse polarizability, guided waves can exist even at low frequencies, having zero cut-off value. Let us consider an example of dipole particles made of short metal sections of conducting wires. The corresponding values of $\mathrm{Re}\{1/\alpha\}$ are frequency-independent but large, as discussed above, and there are no guided-wave solutions. To make the guided-wave solutions possible, the wire dipole particles must be loaded. There are two possibilities: loads which increase capacitance, for example bulk capacitances or high-permittivity coverings of wires, and resonance loads. In the first case, which corresponds to weak frequency dependence of the polarizability (frequency independent in the quasi-static approximation), dispersion curves have the form shown in Figure 4. (in case of capacitive loads the real part of the polarizability is of course positive, so not all the curves can be realized in this way). In case of the resonant loads (inductive loading of short dipole antennas), the left-hand side of the real part of the dispersion equation (8) quickly varies with the frequency. Thus, guided-wave solutions exist only in a very narrow frequency band. This situation can be also visualized using Figure 4, where one should assume that with changing frequency (that is, varying kd), the constant level shown at the curves quickly changes. In this regime, very sharp resonances in the electromagnetic response of the array should be expected.

These results show that very quick spatial variations

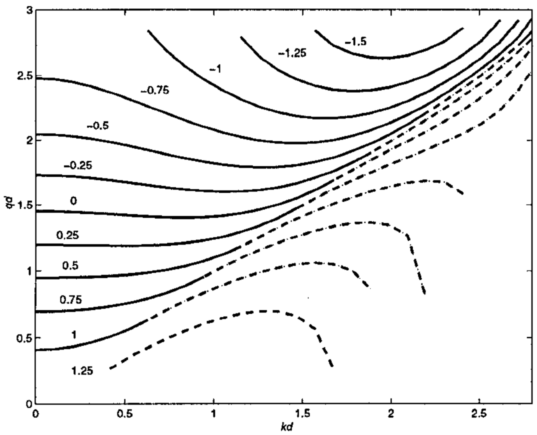

Fig. 4. Dispersion curves for frequency-independent values of $\mathrm{Re}\{1/\alpha\}$. The corresponding normalized values of $\pi\epsilon_0 d^3 \mathrm{Re}\{1/\alpha\}$ are shown near each curve. Curves for $\pi < qd < 2\pi - kd$ can be obtained by reflecting the picture around the line $kd = \pi$. Solid lines correspond to guided-wave solutions, and dash lines show the curves for leaky modes.

of the currents induced on the inclusions (large qd) are possible even at low frequencies (small kd), if the antennas are capacitively loaded. We also observe that the propagation factor q in this case slowly depends on the frequency. This feature can be possibly used to generate current distributions needed to realize superdirective antenna patterns.

IV. Conclusion

Using analytical means, properties of periodical arrangements of small passive scatterers have been explored. Balance between the power scattered by every inclusion and that received by the same inclusion from other inclusions in the array has been discussed and used to determine the region of existence of eigenwaves propagating along the line. It can be shown that both narrow-band and wide-band strong reflection regimes are possible depending on the load impedance of the inclusions. Here, Wood anomalies can be identified from the analysis of the power balance.

In particular, guided eigenmodes have been studied in detail. Guided-wave modes cannot be excited by plane waves in the infinite line, since these modes have propagation constants larger than the wave number in the background medium. That is why in reflection only the classical Wood anomalies related to excitation of higher-order Floquet modes can be seen. Waveguide modes can be excited in arrays of a finite number of inclusions or by finite sources. These modes exist if the particles are loaded by reactive elements (either capacitive or inductive) where electromagnetic energy can be stored. It is well known that superdirective properties of array antennas (or passive arrays, such as an array of grooves in conducting plane [7]) realize if the adjacent elements are located at a distance smaller than $\lambda/2$ and excited out of phase. The same is true for arrays of resonant reflectors, see e.g. [6]. Our results show that waveguide solutions for a line of loaded dipole antennas indeed exist in the range of propagation factors which include this case. Thus, we expect that similar phenomena can be realized in simpler systems with bulk reactive loads. Interesting enough that the required energy storage can be provided by non-resonant capacitive loads, which means that guided modes with very rapid variations of the fields along the line of antennas can exist in wide frequency bands, so that no high-quality resonators are necessary. Naturally, if a narrow-band operation is required, resonance loads can be used.

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